Reprinted from Applied Optics

Accuracies of the diffusion approximation and its similarity relations for laser irradiated biological media

Gilwon Yoon, Scott A. Prahl, and Ashley J. Welch

The accuracy of the diffusion approximation is compared with more accurate solutions for describing light interaction with biological tissues. Generally the diffusion approximation underestimates the light distribution in the surface region, and, for high albedos, it significantly underestimates the fluence rate. This difference is only a few percent for albedos of less than 0.5 due to the dominance of collimated light. As the anisotropy of scattering increases, deviations increase. In general, fluxes can be computed more accurately with the diffusion approximation than fluence rates. For anisotropic scattering, better results can be obtained by simple transforms of optical coefficients using the similarity relations. The similarity relations improve flux calculations, but computed fluence rates have substantial errors for high albedo and the large index of refraction differences at the surface.

I. Introduction

Mathematical descriptions of light absorption and scattering are important in understanding the interaction of laser light with biological media for treatment and diagnostics. One mathematical technique for describing light propagation in biological media is radiative transfer theory. Unfortunately, the general solution is not known, and accurate solutions are limited to simple conditions or slab geometries. Numerical solutions are more versatile but usually require extensive computer memory and time.

On the other hand, the diffusion approximation has been widely used for describing light propagation in biological media especially when scattering dominates absorption.[1-4] In the diffusion approximation, the phase function is represented by the first two terms of the Legendre polynomial expansion. This provides mathematical convenience, and the phase function is characterized by a single anisotropy factor.

In this paper, solutions of the diffusion approximation are examined. However, it is difficult to quantify errors for general conditions since many parameters are involved. Nevertheless, the type and trend of errors can be studied. A 1-D slab geometry has been selected to reduce complexities. Solutions from the discrete ordinate method[5] and from the tables of van de Hulst[6] which are more accurate than solutions using the diffusion approximation are used as references. In addition, simple transforms of the optical coefficients using the similarity relations are examined for improving the accuracy of computed fluxes and fluence rates for anisotropic scattering.

Gilwon Yoon is with National Institute for Health & Medical Research (INSERM, U279), France; S. A. Prahl is with The Laser Center, Academic Medical Center, The Netherlands; and A. J. Welch is with University of Texas at Austin, Biomedical Engineering Program, Austin, Texas 78712.

Received 28 November 1988.
0003-6935/89/122250-06$02.00/0.

II. Diffusion Approximation

The radiative transfer equation is given as[7]

$$\mathbf{s}\cdot\nabla L(\mathbf{r},\mathbf{s}) + \mu_t L(\mathbf{r},\mathbf{s}) = \frac{\mu_s}{4\pi}\int_{4\pi} p(\mathbf{s},\mathbf{s}')L(\mathbf{r},\mathbf{s}')d\omega', \tag{1}$$

where $L(\mathbf{r},\mathbf{s})$ is the radiance (W/cm^{2}sr) at position $\mathbf{r}$ in the $\mathbf{s}$ direction ($\mathbf{s}$ is the directional unit vector), μ_t is the attenuation coefficient (1/cm) defined as the sum of the absorption coefficient μ_a (1/cm), and the scattering coefficient μ_s (1/cm). The phase function $p(\mathbf{s},\mathbf{s}')$ represents scattering contribution from $\mathbf{s}'$ to the $\mathbf{s}$ direction and defined as

$$\left(\frac{1}{4\pi}\right)\int_{4\pi} p(\mathbf{s},\mathbf{s}')d\omega' = 1,$$

where ω denotes the solid angle. It is often convenient to represent the total radiance in terms of the collimated radiance $L_c(\mathbf{r},\mathbf{s})$ and the diffuse radiance $L_d(\mathbf{r},\mathbf{s})$. The collimated radiance which is attenuated by direct absorption and scattering is given by

$$dL_c(\mathbf{r},\mathbf{s})/ds = -\mu_t L_c(\mathbf{r},\mathbf{s}). \tag{2}$$

In the diffusion approximation, the diffuse radiance is approximated as follows:

$$L_d(\mathbf{r,s}) \cong U_d(\mathbf{r}) + \mathbf{F}_d(\mathbf{r}) \cdot \mathbf{s}(3/4\pi), \quad (3)$$

where $U_d(\mathbf{r}) = \int_{4\pi} L_d(\mathbf{r,s})d\omega/4\pi$ is the average radiance, and $\mathbf{F}_d(\mathbf{r}) = \int_{4\pi} L_d(\mathbf{r,s})\mathbf{s}d\omega$ is the diffuse flux vector. Equation (3) may be considered as the first two terms of a Taylor's expansion of L_d.

For a uniform collimated beam normally incident on a slab, the radiance varies only in the propagation direction z. The collimated light expressed by Eq. (2) decays in proportion to $\exp(-\mu_t z)$. By inserting Eqs. (2) and (3) into Eq. (1), the following differential equation in terms of U_d is obtained[7]:

$$\partial^2 U_d(z)/\partial z^2 - 3\mu_a\mu_{tr}U_d(z) = -(3\mu_s\mu_{tr} + 3g\mu_s\mu_t)F_c(z)/4\pi, \quad (4)$$

where $\mu_{tr} = \mu_a + (1 - g)\mu_s$,
g = average cosine angle of phase function (or anisotropy factor) defined as $\int_{4\pi} p(\mathbf{s,s'})\mu \mathbf{d}\omega' / \int_{4\pi} p(\mathbf{s,s'})d\omega'$,
μ = cosine angle of $\mathbf{s}$ with respect to $\mathbf{z}$,
$F_c(z) = L_{inc}\exp(-\mu_t z)$; collimated flux, and L_{inc} is the incident irradiance (W/cm^2).

The collimated flux has a component in the z-direction only. The diffuse radiance can be obtained as follows:

$$L_d(z,\mu) = U_d(z) + (3/4\pi)g\mu_s F_c(z)\mu/\mu_{tr} - \nabla U_d(z)\mu/\mu_{tr}. \quad (5)$$

A. Boundary Conditions

For diffusion approximation, the following conditions at a surface for diffuse light have been implemented[7]:

$$\int_{2\pi,n} L_d(\mathbf{r,s})\mathbf{s} \cdot \mathbf{n} d\omega = 0, \quad (6)$$

where $\mathbf{n}$ is the normal unit vector directed into the medium and the integration is taken over the hemisphere. The diffuse flux entering the medium is set to zero. However, some portion of diffuse light in the tissue is reflected at boundaries where there is a mismatched index of refraction. For biological tissues, values of the index of refraction of ~1.4 have been reported.[8] For mismatched boundary conditions, the total diffuse flux reflected into the medium is part of the outward diffuse flux. This can be implemented as follows:

$$\int_{2\pi,n} L_d(\mathbf{r,s})\mathbf{s} \cdot \mathbf{n} d\omega = \int_{2\pi,-n} R(\mathbf{s} \cdot -\mathbf{n})L_d(\mathbf{r,s})(\mathbf{s} \cdot -\mathbf{n})d\omega, \quad (7)$$

where R is the reflectance whose value can be determined from the index of refraction using the Fresnel equation. For unpolarized light, the reflected radiance with respect to the incident angle $R(\theta_i)$ is represented by the reflected electric fields perpendicular to the plane of incidence $R_\perp$ and parallel to the plane of incidence $R_\parallel$.

$$R(\theta_i) = \frac{1}{2}[R_\perp^2 + R_\parallel^2]$$

$$= \frac{1}{2}\left[\frac{\sin^2(\theta_i - \theta_t)}{\sin^2(\theta_i + \theta_t)} + \frac{\tan^2(\theta_i - \theta_t)}{\tan^2(\theta_i + \theta_t)}\right], \quad (8)$$

where

$$R(0) = \frac{(n_i - n_t)^2}{(n_i + n_t)^2} \quad \text{if } \theta_i = 0,$$

$$R(\theta_i) = 1.0 \quad \text{if } \theta_i > \theta_c = \text{critical angle}.$$

θ_i and n_i represent the incident angle and index of the incident medium, respectively, and θ_t and n_t represent the transmitted angle and index of the transmitted medium.

A similar technique used for Eq. (6) by Ishimaru[7] has been employed to derive the boundary conditions: $\mathbf{F}_d$ in Eq. (3) is separated into normal and tangential components to the surface and placed in Eq. (7). Using Eq. (8), the following expressions for a slab whose thickness is d are obtained:

$$U_d(z) - Ah\partial U_d(z)/\partial_z = -A(g\mu_s/\mu_{tr})F_c(z)/2\pi \text{ at } z = 0,$$
$$U_d(z) + Ah\partial U_d(z)/\partial z = A(g\mu_s/\mu_{tr})F_c(z)/2\pi \text{ at } z = d, \quad (9)$$

where $A = [1 + R_2]/[1 - R_1]$ and $h = 2/3\mu_{tr}$, assuming the medium index is greater than or equal to the index of the environment:

$$R_1 = 2\int_{\mu c}^{1} R(\mu)\mu d\mu + \mu_c^2;$$
$$R_2 = 3\int_{\mu c}^{1} R(\mu)\mu^2 d\mu + \mu_c^3;$$
$$\mu_c = \cos\theta_c.$$

The boundary conditions in Eq. (7) have been applied for the diffusion approximation by Keijzer *et al.*[9] and Star and Marijnissen.[4] For mathematical convenience, they simplified $R(\theta_i)$: Keijzer *et al.* adapted $R(\theta_i) = R(0)$ for $\theta_i \le \theta_c$ and $R(\theta_i) = 1.0$ for $\theta_i > \theta_c$. Star and Marijnissen used exponential functions fitting the Fresnel equation to improve accuracy. In this paper, exact values of $R(\theta_i)$ are computed numerically.

III. Similarity Relations

There can be different sets of μ_a,μ_s and the average cosine angle of the phase function g, which provide similar estimates of radiance in diffusion approximation calculations. These are the so-called similarity relations. The following can be formulated by assuming the same behavior in a diffusion domain[6]:

$$\mu_a' = \mu_a;$$
$$(1 - g')\mu_s' = (1 - g)\mu_s. \quad (10)$$

The primes indicate transformed parameters. In general, there is more than one choice. Each of them, however, distorts the results one way or the other. Notable differences exist among different similarity relations in the surface region, but usually these differences become negligible beyond several optical depths. A practical matter is how to select a specific set of optical coefficients and to study how the computational results are affected.

One motive for using the similarity relations is to transform anisotropic scattering into isotropic scattering. In this case, we have

$$\mu_a' = \mu_a \quad \mu_s' = (1 - g)\mu_s \quad g' = 0. \quad (11)$$

This eases the computational burden, and only the absorption coefficient and effective scattering coeffi-

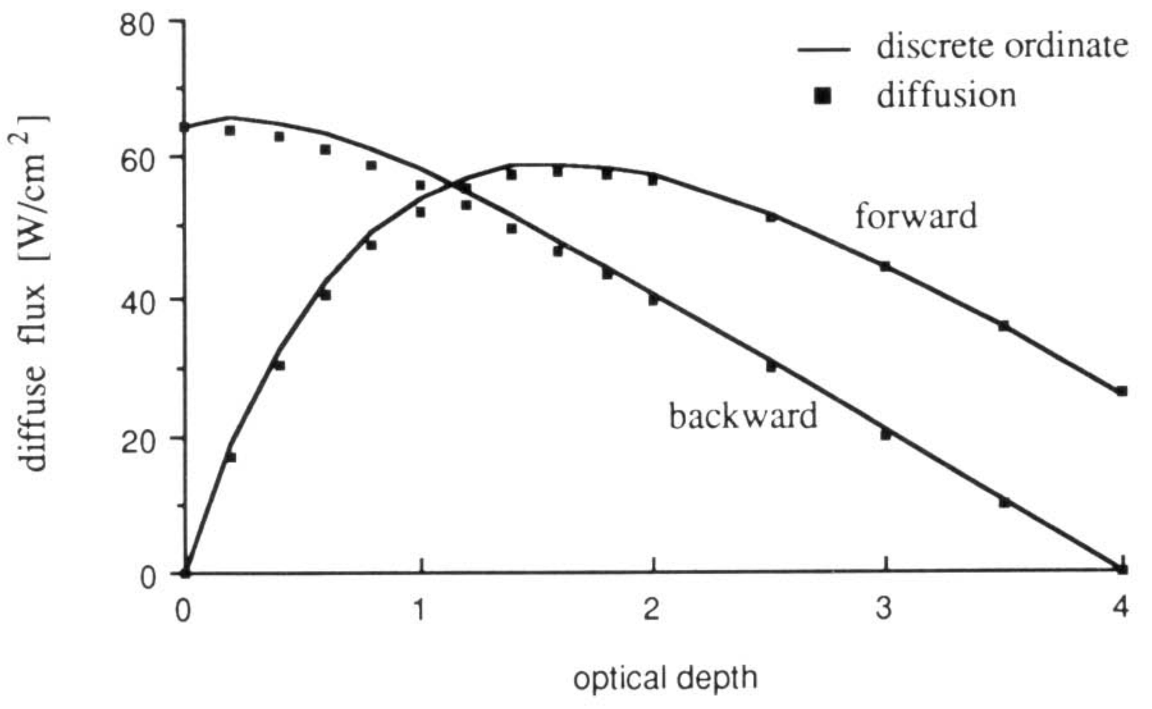

Fig. 1. Forward and backward diffuse fluxes for a uniform collimated irradiance of 100 W/cm^2 are shown as a function of depth. Albedo = 0.99, $g = 0$, sample thickness = 4.0, and matched boundary conditions are assumed. For high albedo and isotropic scattering, there are little differences between diffusion and discrete ordinate solutions.

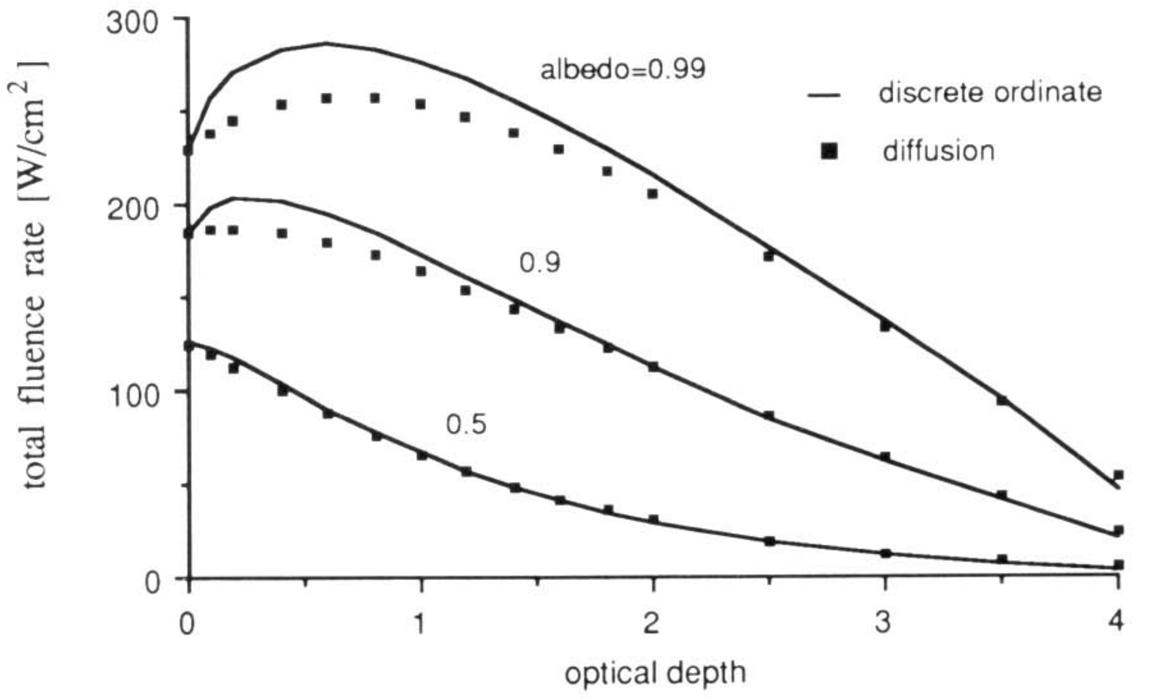

Fig. 2. Total fluence rates computed with the diffusion approximation and the discrete ordinate method with respect to different albedos. Differences are larger for high albedos. $g = 0$, sample thickness = 4.0, matched boundary conditions, and uniformly collimated irradiance of 100 W/cm^2 are assumed.

cient μ_s', are needed to characterize the tissue properties.

Another relation can be formulated with the addition of scattering in the exact forward direction. This is of practical importance since highly forward scattering may be approximated using the delta function. Adding the delta function to the phase function changes the scattering coefficient only. Suppose that the phase function is represented by the delta function and another phase function $p_1(\mathbf{s},\mathbf{s}')$,

$$p(\mathbf{s},\mathbf{s}') = f\delta(\mathbf{s} - \mathbf{s}') + (1 - f)p_1(\mathbf{s},\mathbf{s}'), \tag{12}$$

where f represents the amount of the forward peak. Equation (1) can be formulated with $p_1(\mathbf{s},\mathbf{s}')$ and $(1 - f)\mu_s$ instead of $p(\mathbf{s},\mathbf{s}')$ and μ_s, respectively.

If a phase function is approximated by the above form with $p_1(\mathbf{s},\mathbf{s}')$ in a simple expression, computation is reduced. Accuracy depends on how well the approximate function represents a real function. A method proposed by Joseph *et al.*,[10] which is known as the δ-Eddington approximation, utilizes the above form of phase function: p_1 consists of the first two terms of the Legendre polynomials; i.e., the diffusion phase function and delta function are added to compensate for the highly forward peak since the diffusion function is a poor approximation for anisotropic scattering. For the following Henyey-Greenstein phase function,

$$p_{\text{hg}}(\mathbf{s},\mathbf{s}') = \left(\frac{1}{4\pi}\right)\sum_{n=0}^{\infty}(2n + 1)g^n P_n(\mathbf{s},\mathbf{s}'), \tag{13}$$

where the P_n terms are Legendre polynomials, Joseph *et al.* determined the value of f by setting the δ-Eddington phase function equal to g and representing the second moment as a Henyey-Greenstein function. This yields

$$\mu_a' = \mu_a, \qquad \mu_s' = (1 - g^2)\mu_s, \qquad g' = g/(1 + g). \tag{14}$$

Equation (14) satisfies Eq. (10). The δ-Eddington approximation can be considered as one special case of the similarity relations.

IV. Comparisons with More Accurate Solutions

Computations are for a slab irradiated by a normally incident uniform laser beam. For anisotropic scattering, the Henyey-Greenstein phase function is assumed. Solutions of the diffusion approximation using the original coefficients and the coefficients calculated from Eqs. (11) and (14) are compared with solutions using the discrete ordinate method using twenty-four fluxes[5] and van de Hulst's solutions.[6] Our primary interest is in the following quantities:

forward flux $\int_{2\pi,z} L(\mathbf{r},\mathbf{s})\mathbf{s}\cdot\mathbf{z}d\omega$;
backward flux $\int_{2\pi,-z} L(\mathbf{r},\mathbf{s})\mathbf{s}\cdot(-\mathbf{z})d\omega$;
fluence rate $\int_{4\pi} L(\mathbf{r},\mathbf{s})d\omega$.

However, radiance $L(\mathbf{r},\mathbf{s})$ is also examined. In laser photothermal reactions, the local rate of heat generation given by ($\mu_a \times$ fluence rate) is required to calculate temperatures within laser irradiated tissue.

A. Isotropic Scattering Case

For isotropic scattering ($g = 0$), the optical coefficients given by Eqs. (11) and (14) are the same as the original coefficients. First, we examine the case where the diffusion approximation is known to be accurate, i.e., high albedo, isotropic scattering, and matched boundary conditions. The albedo is defined as μ_s/μ_t. Fairly accurate 1-D solutions are obtained with the diffusion approximation in terms of fluxes, i.e., forward, backward, and net fluxes. For example, diffuse forward and diffuse backward fluxes show little differences compared with discrete ordinate solutions (Fig. 1). In this illustration, optical depth is defined as $\tau = \mu_t \times$ geometrical depth, and the following parameters are used: albedo 0.99, collimated irradiance 100 W/cm^2, an optical thickness of 4.0. For both models, the collimated fluxes are the same and decrease according to exp(−optical depth).

Usually, larger errors are associated with fluence rate calculations (or the rate of heat generation) than with estimates of flux. This is illustrated in Fig. 2 where total fluence rates computed with diffusion and discrete ordinate models are plotted as a function of optical depth. The fluence rates at the front surface computed with both models are almost the same, but

Reprinted from Applied Optics

Accuracies of the diffusion approximation and its similarity relations for laser irradiated biological media

Gilwon Yoon, Scott A. Prahl, and Ashley J. Welch

The accuracy of the diffusion approximation is compared with more accurate solutions for describing light interaction with biological tissues. Generally the diffusion approximation underestimates the light distribution in the surface region, and, for high albedos, it significantly underestimates the fluence rate. This difference is only a few percent for albedos of less than 0.5 due to the dominance of collimated light. As the anisotropy of scattering increases, deviations increase. In general, fluxes can be computed more accurately with the diffusion approximation than fluence rates. For anisotropic scattering, better results can be obtained by simple transforms of optical coefficients using the similarity relations. The similarity relations improve flux calculations, but computed fluence rates have substantial errors for high albedo and the large index of refraction differences at the surface.

I. Introduction

Mathematical descriptions of light absorption and scattering are important in understanding the interaction of laser light with biological media for treatment and diagnostics. One mathematical technique for describing light propagation in biological media is radiative transfer theory. Unfortunately, the general solution is not known, and accurate solutions are limited to simple conditions or slab geometries. Numerical solutions are more versatile but usually require extensive computer memory and time.

On the other hand, the diffusion approximation has been widely used for describing light propagation in biological media especially when scattering dominates absorption.[1–4] In the diffusion approximation, the phase function is represented by the first two terms of the Legendre polynomial expansion. This provides mathematical convenience, and the phase function is characterized by a single anisotropy factor.

In this paper, solutions of the diffusion approximation are examined. However, it is difficult to quantify errors for general conditions since many parameters are involved. Nevertheless, the type and trend of errors can be studied. A 1-D slab geometry has been selected to reduce complexities. Solutions from the discrete ordinate method[5] and from the tables of van de Hulst[6] which are more accurate than solutions using the diffusion approximation are used as references. In addition, simple transforms of the optical coefficients using the similarity relations are examined for improving the accuracy of computed fluxes and fluence rates for anisotropic scattering.

Gilwon Yoon is with National Institute for Health & Medical Research (INSERM, U279), France; S. A. Prahl is with The Laser Center, Academic Medical Center, The Netherlands; and A. J. Welch is with University of Texas at Austin, Biomedical Engineering Program, Austin, Texas 78712.

Received 28 November 1988.
0003-6935/89/122250-06$02.00/0.

II. Diffusion Approximation

The radiative transfer equation is given as[7]

$$\mathbf{s}\cdot\nabla L(\mathbf{r},\mathbf{s}) + \mu_t L(\mathbf{r},\mathbf{s}) = \frac{\mu_s}{4\pi}\int_{4\pi} p(\mathbf{s},\mathbf{s}')L(\mathbf{r},\mathbf{s}')d\omega', \tag{1}$$

where $L(\mathbf{r},\mathbf{s})$ is the radiance (W/cm^2sr) at position $\mathbf{r}$ in the $\mathbf{s}$ direction ($\mathbf{s}$ is the directional unit vector), μ_t is the attenuation coefficient (1/cm) defined as the sum of the absorption coefficient μ_a (1/cm), and the scattering coefficient μ_s (1/cm). The phase function $p(\mathbf{s},\mathbf{s}')$ represents scattering contribution from $\mathbf{s}'$ to the $\mathbf{s}$ direction and defined as

$$\left(\frac{1}{4\pi}\right)\int_{4\pi} p(\mathbf{s},\mathbf{s}')d\omega' = 1,$$

where ω denotes the solid angle. It is often convenient to represent the total radiance in terms of the collimated radiance $L_c(\mathbf{r},\mathbf{s})$ and the diffuse radiance $L_d(\mathbf{r},\mathbf{s})$. The collimated radiance which is attenuated by direct absorption and scattering is given by

$$dL_c(\mathbf{r},\mathbf{s})/ds = -\mu_t L_c(\mathbf{r},\mathbf{s}). \tag{2}$$

In the diffusion approximation, the diffuse radiance is approximated as follows:

C. Mismatched Boundary Conditions

Biological tissues usually have higher indices of refraction than the air or the surrounding fluid. Light is internally reflected and trapped, which causes higher intensity compared with the matched boundaries. The larger the tissue index is, the higher fluence rates are generated.

In Fig. 5, the fluence rates for two different g's of 0.0 and 0.8 are calculated for an index of refraction of 1.4 to represent the biological tissue and an albedo of 0.99. The increase in the fluence rate due to internal reflection is very significant especially for high albedos: for albedo = 0.99 and $g = 0$, the fluence rate at the front surface increases by more than a factor of 2 (see the discrete ordinate solutions in Figs. 2 and 5). However, when the δ-diffusion approximation is used, the fluence rate increases by only 80%. It is interesting to note that the largest underestimation by the diffusion approximation is observed in the subsurface region for the matched boundaries and at the front surface for the mismatched boundaries.

Comparison of the isotropic scattering transform and forward scattering transform is also featured in Fig. 5. For the diffusion approximation, the fluence rates for $g = 0.8$ are computed using Eq. (14). For $g = 0.8$, light penetrates deeper in the tissue due to forward scattering. Errors of the diffusion solutions are high for all optical depths. For albedos smaller than 0.5, differences in the fluence rates between diffusion and discrete ordinate solutions are very small for either mismatched boundary conditions or anisotropic scattering. The diffusion approximation as well as other models do not show large differences from Beer's law when absorption is dominant.[11]

D. Radiance

The diffusion approximation uses the first two terms of Legendre polynomials to represent radiance. Therefore, estimates of radiance are accurate in the diffusion region, i.e., beyond several optical depths or at least a few optical depths. Estimates of radiance are not accurate at the boundaries or for thin samples where radiances are highly anisotropic. For example, for matched boundary conditions and assuring that no diffuse flux enters the tissue [see Eq. (6)], negative reflectance values are shown in Fig. 3 for high g. The exact boundary conditions for the radiative transfer equation are no radiance entering a medium for the matched indices. Each ray entering the tissue must be determined from the Fresnel equation for the mismatched indices. In the diffusion approximation, the boundary conditions for each ray cannot be implemented.

For anisotropic scattering, it is more difficult to predict the radiance profiles. For example, angular distributions of light intensity computed using the original coefficients and Eqs. (11) and (14) are illustrated in Fig. 6. Radiances at an optical depth of 3.0 are compared with discrete ordinate solutions. An albedo = 0.99, $g = 0.8$, optical thickness of sample = 4.0, and matched boundary conditions are assumed for computation. In the figure 0° indicates the direction of beam propagation, and 180° is the opposite direction. For irradiation with a uniform collimated beam most diffuse light at an optical depth of 3.0 is confined within the ±45° cone in the forward direction (discrete ordinate solutions, solid line in Fig. 6). On the other hand, the three diffusion solutions have flattened profiles. The collimated flux has intensity only in the direction of beam propagation. It is reduced from 100 to 5.0 W/cm^2, which is smaller than the diffuse radiance at this optical depth. The collimated intensity of the discrete ordinate method is much smaller than the diffuse intensity at 0°.

Among the three diffusion calculations described in this paper, the original coefficients yield higher diffuse radiance but the smallest collimated radiance. Trans-

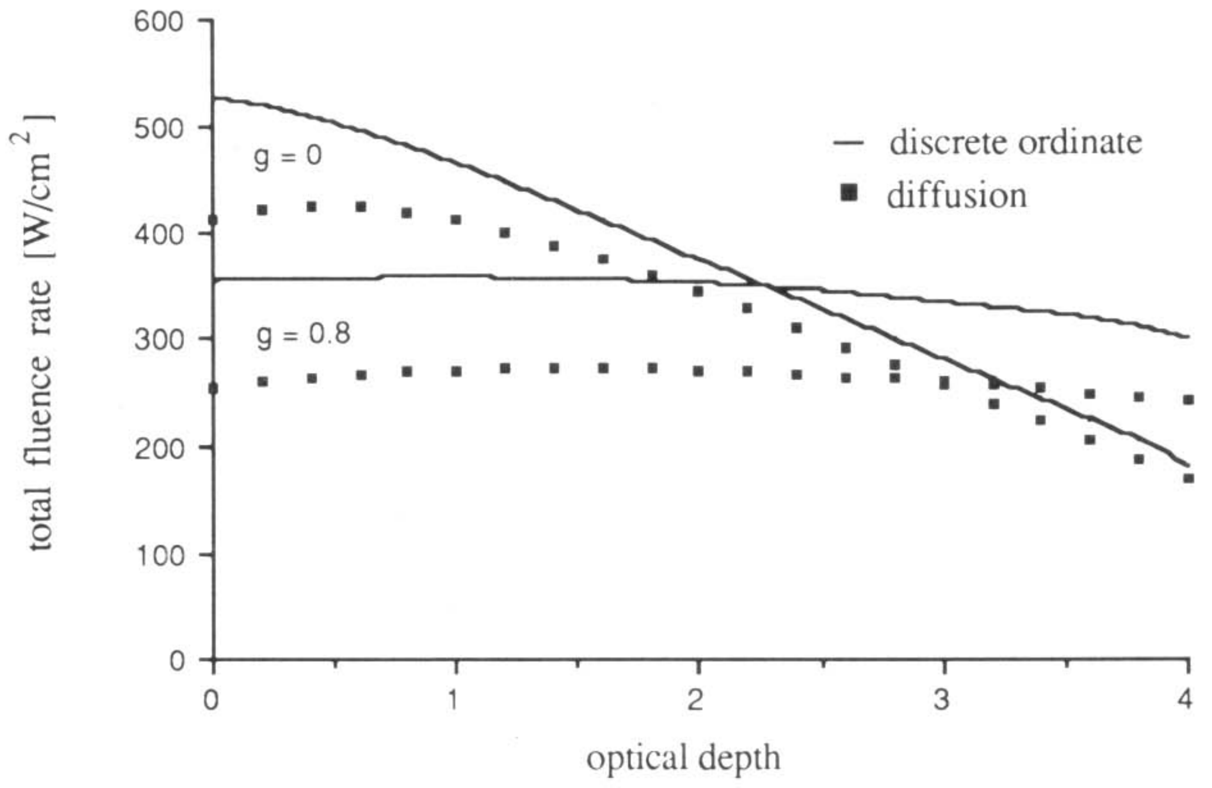

Fig. 5. Total fluence rates for a tissue index of 1.4 are computed from diffusion and discrete ordinate models. Sample thickness = 4.0, albedo = 0.99, and a uniform irradiance of 100 W/cm^2 are assumed. For $g = 0.8$ Eq. (14) is used to transform the coefficients. For diffuse light, the boundary conditions in Eq. (9) are implemented. For collimated light, the external reflection of 2.8% is considered only at the front surface.

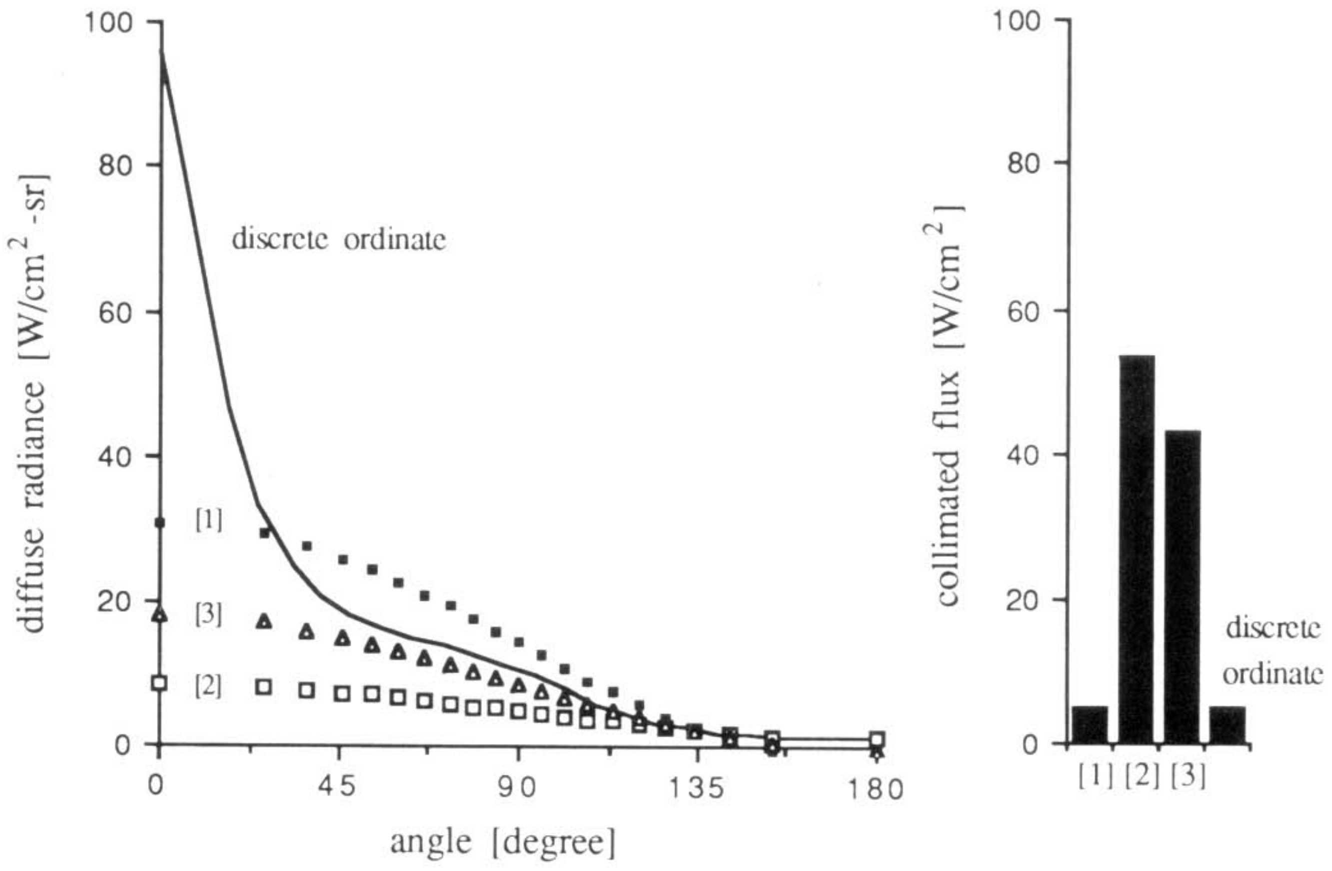

Fig. 6. Diffuse radiance (W/cm^2 sr) and collimated flux (W/cm^2) at an optical depth of 3.0. Albedo = 0.99, $g = 0.8$, sample thickness = 4.0, and matched boundary conditions are assumed: [1] orginal coefficients ($\mu_a = 0.01, \mu_s = 0.99, g = 0.8$); [2] $\mu_a' = 0.01, \mu_s' = 0.198$, and $g' = 0$ using Eq. (11); [3] $\mu_a' = 0.01, \mu_s' = 0.356$, and $g' = 0.444$ using Eq. (14) are compared with discrete ordinate solutions.

forms of the optical properties reduce diffuse light and increase collimated light for the compensation of forward peaks; i.e., Eqs. (11) and (14) reduce effective scattering. Using Eq. (11), anisotropic scattering is converted into isotropic scattering and the effective scattering coefficient becomes smaller by a factor of $1 - g$. Therefore, there is less diffuse light and lower attenuation of collimated light, i.e., $\exp(-\mu_a - \mu_s')$. For Eq. (14), the effective scattering reduces by a factor of $1 - g^2$. Equation (14) seems to be better than either of the other diffusion solutions when the total radiance of both diffuse and collimated light is considered. The original coefficients hardly represent a forward peak. In general, all three solutions based on the diffusion model are poor approximations of the angular distribution of light except in the diffusion regions away from the source and boundaries.

V. Conclusion

Accuracies of the models based on the diffusion approximation have been examined in a slab geometry irradiated by a uniform normal beam. Similar trends, even though the magnitudes may not be of the same order, are also observed in other geometries including the 2-D axial symmetry model. For high albedos and isotropic scattering, fluxes are estimated accurately by the diffusion approximation, but lower fluence rates (or rates of heat generation) are computed in the surface region. For low albedos, fluxes are less accurate, but the total fluence rates are better estimated due to the dominance of collimated light. In general, the diffusion approximation estimates low reflection and high transmission for forward scattering and is not recommended when g is larger than ~0.6. The analysis using the diffusion approximation for highly anisotropic scattering can be very misleading.

For anistropic scattering, simple parametric transforms using the similarity relations provide better results. The similarity relations provide many improvements in terms of fluxes. The δ-Eddington approximation of Eq. (14) yields the best flux estimates for all conditions. In predicting the fluence rate, the similarity transform of Eq. (11) is no better than the original coefficients. Equation (14) is better than the other two in estimating fluence rates, but substantial errors for high albedos and large index differences still exist.

Some methods are available for improving the diffusion approximation. For example, the diffusion approximation for large absorption has been studied,[12] but its application is restricted to diffuse incidence. The P_N approximation is an extension of the diffusion approximation for the general function, but it encounters computational complexities for highly anisotropic scattering and 2- or 3-D geometries. Geometrical flexibility and mathematical convenience are advantages of using the diffusion approximation. These merits have to be compromised with decreased accuracy.

This work was funded in part by the Office of Naval Research under contract N14-86-0875.

The authors would like to thank Willem Star for his very useful remarks.

Gilwon Yoon is on leave from the Utah Laser Institute of the University of Utah.

References

1. R. A. J. Groenhuis, H. A. Ferwerda, and J. J. T. Bosch, "Scattering and Absorption of Turbid Materials Determined from Reflection Measurements," Appl. Opt. **22,** 2456–2467 (1983).
2. S. L. Jacques and S. A. Prahl, "Modeling Optical and Thermal Distribution in Tissue During Laser Irradiation," Lasers Surg. Med. **6,** 494–503 (1987).
3. J. M. Steinke and A. P. Shepherd, "Diffusion Model of the Optical Absorbance of Whole Blood," J. Opt. Soc. Am. A **5,** 813–822 (1988).
4. W. M. Star and J. P. A. Marijnissen, "Calculating the Response of Isotropic Light Dosimetry Probes as a Function of the Tissue Refractive Index," Appl. Opt. **28,** 2288–2291 (1989).
5. W. G. Houf and F. P. Incropera, "An Assessment of Techniques for Predicting Radiation Transfer in Aqueous Media," J. Quant. Spectrosc. Radiat. Transfer **23,** 101–115 (1980).
6. H. C. van de Hulst, *Multiple Light Scattering: Tables, Formulas, and Applications,"* (Academic, New York, 1980), Vols. 1, 2.
7. A. Ishimaru, *Wave Propagation and Scattering in Random Media* (Academic, New York, 1978), Vol. 1.
8. F. P. Bolin, L. E. Preuss, R. C. Taylor, and R. J. Ference, "Measurement of the Index of Refraction of Mammalian Tissue," *OSA Annual Meeting, 1988 Technical Digest Series, Vol. 11* (Optical Society of America, Washington, DC, 1988), paper TH04.
9. M. Keijzer, W. M. Star, and P. R. M. Storchi, "Optical Diffusion in Layered Media," Appl. Opt. **27,** 1820–1824 (1988).
10. J. H. Joseph, W. J. Wiscombe, and J. A. Weinman, "The Delta-Eddington Approximation for Radiative Flux Transfer," J. Atm. Sci. **33,** 2452–2459 (1976).
11. A. J. Welch, J. A. Pearce, K. R. Diller, G. Yoon, and W. F. Cheong, "Heat Generation in Laser Irradiated Tissue," ASME J. Biomech. Eng. **111,** 62–68 (1989).
12. W. E. Meador and W. R. Weaver, "Diffusion Approximation for Large Absorption in Radiative Transfer," Appl. Opt. **18,** 1204–1208 (1979).